Geometry, Quarter 3, Unit 3.1

Explaining Volume Formulas and Modeling Geometric Shapes

Overview

Number of instructional days: 11 (1 day = 45–60 minutes)

Content to be learned

- Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone, using dissection arguments, Cavalieri's Principle, and informal limit arguments.
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects.
- Identify three-dimensional objects generated by rotations of two-dimensional objects.
- Describe objects by exploring geometric shapes and their measures.
- Use geometric shapes and their properties to describe objects.
- Define concepts of density based on area and volume of geometric shapes.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Draw geometric shapes to predict the circumference of each object.
- Discuss and compare the differences of two-dimensional and three-dimensional shapes.

Construct viable arguments and critique the reasoning of others.

- Define conic formulas in modeling situations.
- Compare the effectiveness of circumference of a circle and the area of a circle.

Model with mathematics.

- Use graph paper to illustrate the rotation of geometric shapes.
- Use formulas to determine measures of geometric shapes.

Essential questions

- How would you make an informal argument for the formula for the circumference of a circle? The area of a circle? The volume of a cylinder? The volume of a pyramid? The volume of a cone?
- What are the similarities of and differences between two-dimensional and three-dimensional objects?
- What is meant by a two-dimensional cross section of a three-dimensional shape?
- How would you describe the rotation of a two-dimensional figure?
- How can a coordinate grid be used to model and describe geometric shapes?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometric Measurement and Dimension — G-GMD

Explain volume formulas and use them to solve problems

G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*.

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects

G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry — G-MG

Apply geometric concepts in modeling situations

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

G-MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In kindergarten, students identified and described three-dimensional shapes. In first grade, students confirmed that a three-dimensional shape is a closed object. In second grade, students students studied shapes containing more sides. Students portioned circles into two, three, or four equal shares. In third grade, students understood that shapes were in different categories. In fourth grade, students classified two-dimensional figures based on the presence or absences of parallel or perpendicular lines. In fifth grade, students classified two-dimensional figures in a hierarchy based on properties. In sixth grade, students solved real-world mathematical problems involving area, surface area, and volume. In seventh grade, students described two-dimensional figures that resulted from cutting three-dimensional figures with a plane. In eighth grade, students understood that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, or transformations.

Current Learning

In this unit, students experiment with two-dimensional and three-dimensional objects to explain informally circumference, area, and volume formulas. They use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. They identify the shapes of two-dimensional cross-sections of three-dimensional objects, and they identify three-dimensional objects generated by rotations of two-dimensional objects. Students describe objects by exploring geometric shapes and their measures, they use geometric shapes and their properties to describe objects, and they define concepts of density based on area and volume of geometric shapes.

Future Learning

In algebra 2, students will extend their previous work with trigonometric ratios and circles in geometry to explore the effects of transformations on graphs of diverse functions. In statistics, students will interpret differences in shape, center, and spread in the context of data sets.

Additional Findings

Add info.

Geometry, Quarter 3, Unit 3.2

Using Definitions and Constructing Figures Inscribed in a Circle

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Define angle, circle, perpendicular line, parallel line, and line segment.
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- Prove theorems about circles using constructions.
- Explore the relationship between an equilateral triangle and a square inscribed in a circle.
- Experiment the relationship between perpendicular and parallel lines.

Mathematical practices to be integrated

Model with mathematics.

- Use constructions with lines and circles.
- Use a graphing calculator to construct a circle on a coordinate grid.

Use appropriate tools strategically.

- Use dynamic geometric software to view polygons inscribed in circles.
- Use transparencies to define circles, perpendicular/parallel lines, segments from a given point.

Attend to precision.

- Demonstrate algebraically inscribed circles lengths.
- Illustrate how to use a protractor to find angle measures within a circle.

Look for and express regularity in repeated reasoning.

- Use definitions to evaluate inscribed circles.

Essential questions

- What methods can be used to construct an equilateral triangle in a circle? A square? A hexagon?
- What is the precise definition of a line segment within a circle?
- How can you use inscribed circles to solve real-world problems?
- What are characteristics of perpendicular and parallel lines?

Written Curriculum

Common Core State Standards for Mathematical Content

Congruence	G-CO

Experiment with transformations in the plane

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc.~~

Make geometric constructions [*Formalize and explain processes*]

G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students were introduced to shapes. In first grade, students are drew shapes to define their attributes. In second grade, students identified shapes by their number of sides. In third grade, students understood shapes in different categories. In fourth grade, students classified two-dimensional figures based on the presence or absences of parallel or perpendicular lines. In fifth grade, students drew geometric shapes and lines on a coordinate plane. In sixth grade, students used the area of triangles and polygons to solve real-world math problems. In seventh grade, the students drew, constructed, and described geometrical figures and the relationships between them. In the eighth grade, students understood the formulas for the volumes of geometric shapes to solve real-world mathematical problems.

Current Learning

In this unit, students prove basic theorems about circles relating to tangent lines, radius, inscribed angle theorem, and theorems pertaining to circle parts. They also construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle

Future Learning

In algebra 2, students will use their previous work with trigonometric ratios and circles in geometry to model periodic phenomena. In precalculus students will use their knowledge of geometry to explain volume formulas and use them to solve problems. Also, they will prove and apply trigonometric identities.

Additional Findings

In this unit, students will expand their knowledge of thermos in a variety of formats to solve problems about triangles and other polygons. [Need to clarify what this means. BH] They will also apply reasoning to complete geometric constructions and explain why they work.

Geometry, Quarter 3, Unit 3.3

Using Theorems of Circles

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Prove that circles are similar.
- Describe the relationship between inscribed and circumscribed angles.
- Construct a line tangent to a circle from a point outside the circle.
- Prove properties of quadrilaterals within a circle.
- Define the relationship among the radii and chords of a circle.
- Prove that the radius of a circle is perpendicular to a line tangent to the circle.

Mathematical practices to be integrated

Model with mathematics.

- Construct circles on a coordinate plane to show similarity.
- Use constructions with circles involving tangent lines, radius, and chords.

Use appropriate tools strategically.

- Use a graphing calculator to draw inscribed circles.
- Use a compass to construct a tangent line to a circle from a given point outside a circle.

Attend to precision.

- Use postulates and theorems to show that circles are similar in a paragraph proof.
- Demonstrate the relationship among inscribed angles, radii, and chords of a circle.

Look for and express regularity in repeated reasoning.

- Use definitions to prove theorems of circles.

Essential questions

- What is a definition of circumscribed angles and inscribed angles?
- What method is best used to construct a circumscribed circle on a coordinate plane?
- What criteria would be used to prove that circles are similar?

Written Curriculum

Common Core State Standards for Mathematical Content

Circles	G-C

Understand and apply theorems about circles

G-C.1 Prove that all circles are similar.

G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle*.

G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G-C.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

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Clarifying the Standards

Prior Learning

In kindergarten, students were introduced to shapes. In first grade, students partitioned circles into equal parts. In second grade, students continued to partition circles but also labeled each part as 1/2, 1/3, 1/4… In fourth grade, students identified two-dimensional figures involving special triangles. In seventh grade, students solved real-world mathematical problems involving area, volume, and surface area of geometric shapes. In eighth grade, students used and understood formulas for volume for cones, cylinders, and spheres to solve real-world problems.

Current Learning

In this unit, students prove that circles are similar, and they describe the relationship between central, inscribed, and circumscribed angles. Students also prove properties of quadrilaterals inscribed inside circles.

Future Learning

In Algebra 2, students will use the coordinate plane to extend trigonometry to model periodic phenomena. They will also explore the effects of transformations on graphs of diverse functions in order to abstract the principles of transformations on graphs that have the same effect regardless of the type of function.

Additional Findings

In this unit, students will prove basic theorems about circles with particular attention to perpendicular and inscribed angles to see similarity in circles and as an application of triangle congruency.